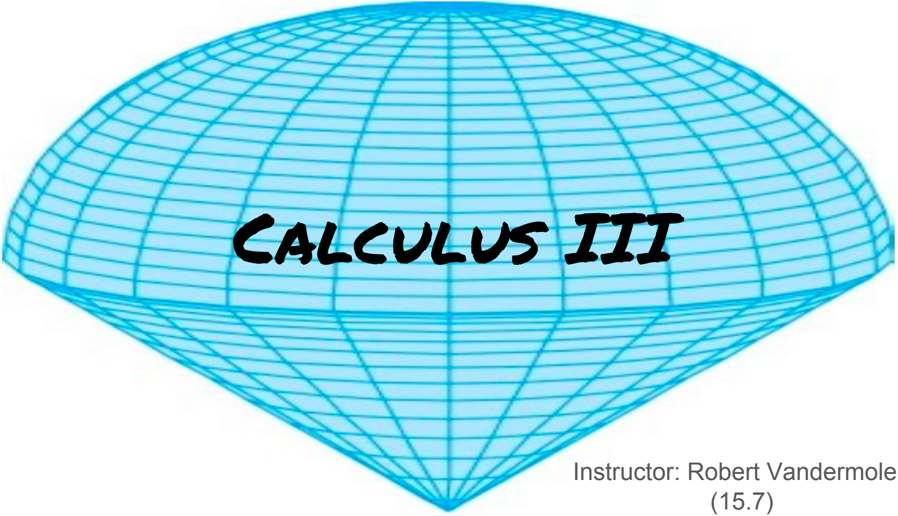
15.7 Lecture: Cylindrical Coordinates

Jeremiah Southwick
(And Robert Vandermolen)

Spring 2019

Links

Robert's slides can be found here:
http://people.math.sc.edu/robertv/teaching.html
The 15.7 slides can be found here:
https://docs.google.com/presentation/d/1V_
CtHJvjz4-etPuIfNYLhpO8ohjmrU6nbetc9xTzqiU



Calculus III

Instructor: Robert Vandermolen
(15.7)

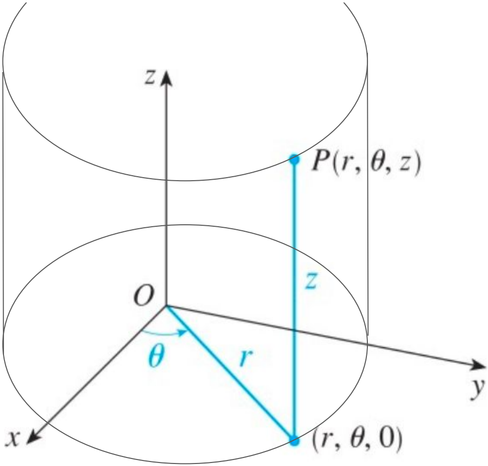
Cylindrical Coordinates!

Just as in double integrals, when we found at times it was easier to calculate the integral in polar coordinates, we have a 3-dimensional version of polar coordinates for the triple integral, which is called Cylindrical Coordinates:

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

$$z = z$$



It is just polar coordinates, but now with a z

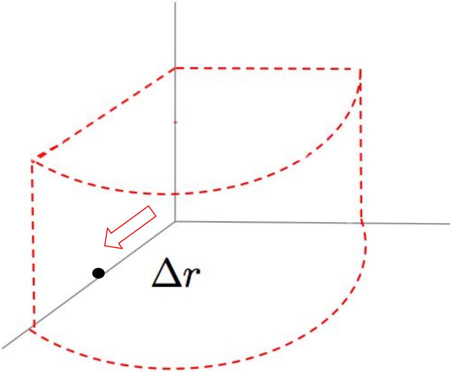
Cylindrical Coordinates!

Just as in double integrals, when we found at times it was easier to calculate the integral in polar coordinates, we can also convert our triple integrals to cylindrical coordinates

$$\iiint_E f(x, y, z)\, dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta,\, r\sin\theta)}^{u_2(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$$

Cylindrical Coordinates!

The picture to keep in our mind while integrating using cylindrical coordinates is the following:



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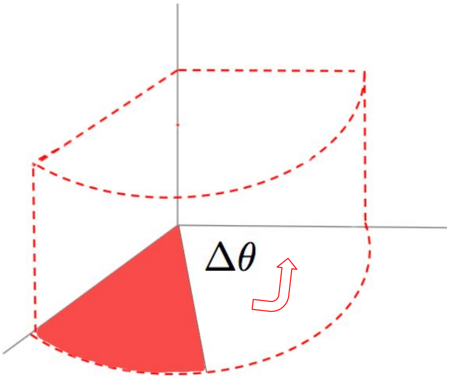

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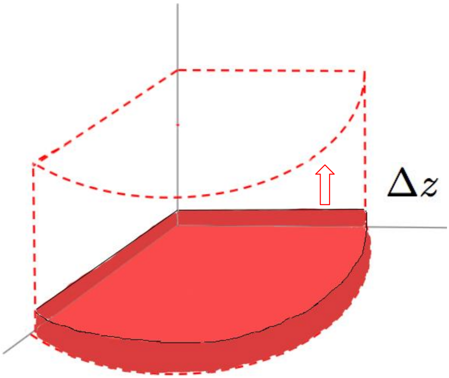
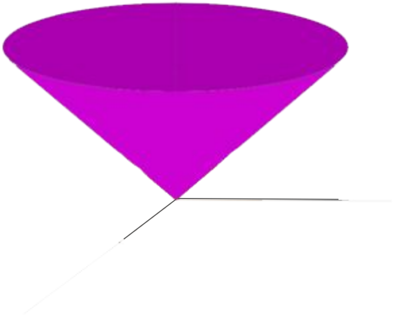


EXAMPLE:

Use Cylindrical Coordinates to make the following integral easier to solve.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\;dz\;dy\;dx$$

r^2 $\quad$ $r\,dz\,dr\,d\theta$



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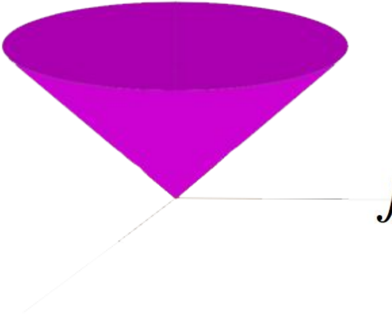

$$\int_{?}^{?}\int_{?}^{?}\int_{?}^{?}(r^2)\,r\,dz\;dr\;d\theta$$

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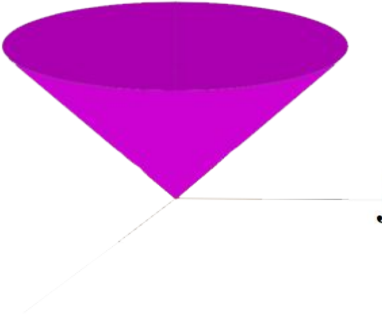
Look this doesn't change....



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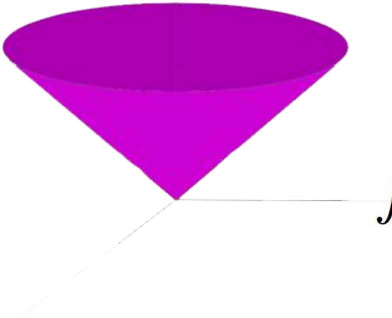
So write this in cylindrical coordinates (just polar)



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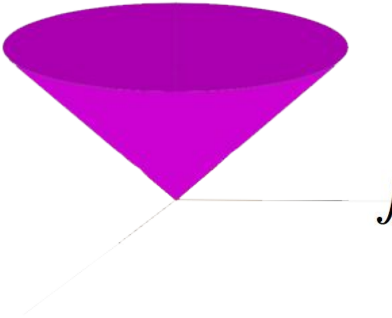

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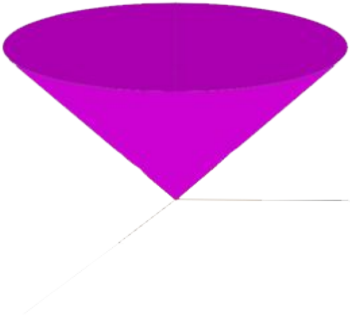

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$$\int_{?}^{?}\int_{?}^{?}\int_{r}^{2}(r^2)r\,dz\,dr\,d\theta$$

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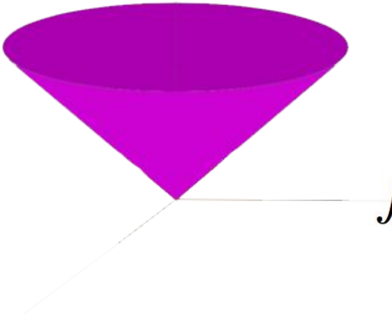
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\ dz\ \boxed{dy\ dx}$$

These DO CHANGE!!!!!



$$\int_{?}^{?}\int_{?}^{?}\int_{r}^{2} (r^2) r dz\ \boxed{dr\ d\theta}$$

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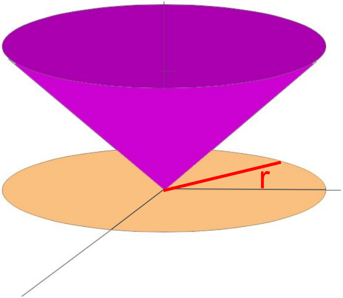

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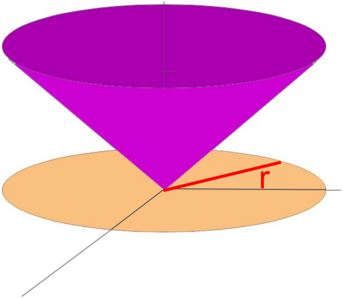

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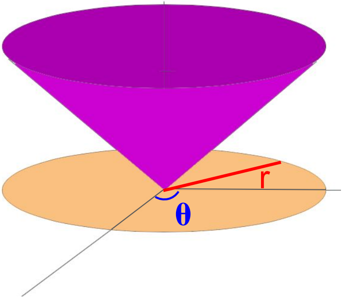

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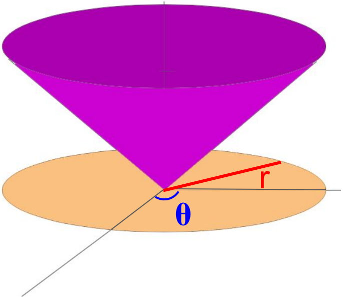
$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} \left(x^2+y^2\right)\ dz\ dy\ dx$$

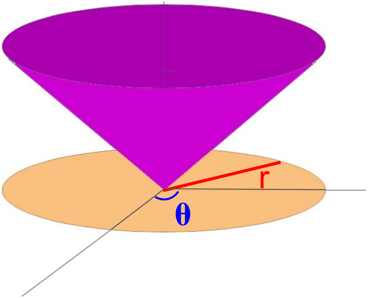


$$\int_0^{2\pi}\int_0^{2}\int_r^{2} (r^2) r dz\ dr\ d\theta$$

Example:

Use Cylindrical Coordinates to make the following integral easier to solve.

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\ dz\, dy\, dx$$



$$= \int_0^{2\pi} \int_0^2 \int_r^2 (r^2)\, r\ dz\ dr\ d\theta$$

Now You Try!

Practice finding these "iterated" integrals

- $$\int_0^{\pi/2} \int_0^3 \int_0^{e^{-r^2}} r \, dz \, dr \, d\theta$$
- $$\int_0^{2\pi} \int_0^{\sqrt{3}} \int_0^{3-r^2} r \, dz \, dr \, d\theta$$